



RESEARCH ARTICLE

OUTPUT-ORIENTED AND INPUT-ORIENTED MEASURES OF PRODUCTION EFFICIENCY IN THE NIGERIAN AGRICULTURE: A TIME SERIES COMPARATIVE ANALYSIS

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ARTICLE DETAILS

Article History:

Received 26 June 2025

Revised 19 July 2025

Accepted 29 August 2025

Available online 30 September 2025

ABSTRACT

This study examines production efficiency in Nigerian agriculture over 62 years (1960–2021) using a stochastic production frontier model and the Cobb-Douglas functional form. The analysis focused on output- and input-oriented efficiency to measure the potential for improving outputs without increasing input usage. An annual, secondary, time series data from the Nigerian agricultural industry, spanning 1960 to 2021 were used. Data sourced from the Food and Agriculture Organisation (FAO) and the National Bureau of Statistics (NBS) databases. Key variables analysed include the sector's agricultural gross domestic product (GDP), fertiliser usage measured in kilograms, agricultural labour quantified in man-days, and the number of tractors available within the sector. The study employed the Augmented Dickey-Fuller (ADF) test, confirming that all variables are integrated of order one (1), thus fit for co-integration analysis. The VAR lag selection criteria identified lag 2 as best. Johansen cointegration tests (trace and maximum eigenvalue) revealed the existence of at least one cointegrating relationship, indicating a long-run equilibrium among the variables. The Stochastic Frontier Production Function (SFPF) estimation revealed a technically efficient agricultural sector with a 78% output-oriented technical efficiency (TE) and 22% inefficiency, suggesting potential for increased productivity without input augmentation. The returns to scale (RTS) of 0.9226 confirm decreasing returns, implying reduced output gains from proportional input increases. Economic analysis further revealed that the sector is only 58% economically efficient (EE), with a cost inefficiency of 42%, indicating room for substantial cost reduction. The cost efficiency (CE) of 1.7115 supports this inefficiency, expressing that actual costs exceed minimum required costs by 71%. Input-oriented measures revealed lower efficiency scores compared to output-oriented ones. Input-oriented TE and EE were 74.6% and 53.8%, respectively, indicating relatively less efficiency when adjusting inputs than outputs. Cost inefficiency under the input route is particularly high at 85.7%. Output-oriented strategies yield higher efficiency measures than input-oriented strategies, validating that policy and production strategies should focus on maximising output from given resources, rather than simply minimising inputs.

KEYWORDS

Stochastic frontier, Return to scale, cost function, Scale effect, Technical efficiency, inefficiency.

1. INTRODUCTION

Nigeria is a vast agricultural country endowed with substantial natural resources, including 68 million hectares of arable land, freshwater resources, 12 million hectares, 960 kilometres of coastline, and ecological diversity that permits a wide variety of agricultural commodities (Arokoyo, 2012). Despite the Agricultural endowments, there has been a gradual decline in agriculture's contribution to the nation's economy (Ekpo and Umoh, 2012). The trend in the share of agriculture in GDP exhibits significant variation and a long-term decline, decreasing from 60% in the early 1960s to 48.8% in the 1970s, then to 22.2% in the 1980s, and finally 21.9% in 2019 (NBS, 2014; World Bank, 2020). Despite the optimistic paradigms of the 60s and 80s, which envisioned agriculture as a growth engine, these expectations have not yet been met (Johnson and Mellor, 1961; Hayemi and Rutten, 1985). The sector's productivity has not only stagnated but declined (Njoku, 2014; NBS, 2014). These challenges

highlight the agricultural industry as a critical area for further research.

As opined that paying attention to productivity gains from a more efficient use of existing resources is more justified than adopting new technology (Squires and Tobor, 1991). Hence, the shortcomings in efficiency (inefficiency) in the sector mean that output can be increased without requiring additional conventional inputs through output-oriented or input-oriented or non-conventional inputs, such as enhanced input quality, market access and others. For this reason, empirical measures of production efficiency (technical efficiency, allocative efficiency, economic efficiency, and cost efficiency) are necessary. There are few empirical studies on production efficiency in Nigerian agriculture based on time series analysis. There is a myriad of cross-sectional studies based on one crop or the other. The sector has not been covered as an economic entity.

To examine farmers operational effectiveness, production efficiency is regularly employed. Efficiencies are often divided into two concepts;

Quick Response Code



Access this article online

Website:
www.fabm.org.my

DOI:
10.26480/fabm.02.2025.100.106

technical efficiency (output-oriented efficiency) and allocative efficiency (input-oriented efficiency). Allocative efficiency is the capability of a farm to use inputs in right proportions considering their individual prices and technology (i.e. obtaining optimal output or profits with the least cost of production). Technical efficiency, conversely, is the competency of a farming unit to produce optimum output level with respect to input level (Farrell, 1957). Evaluating output-oriented technical efficiency, the inputs are from external sources given, and the objective is to enhance output as the only choice variable

1.1 The Statement of the Problem

The major macroeconomic objective of any nation is to achieve economic development via economic growth. The literature highlighted the fact that an effective economic development strategy depends critically on production efficiency in the agricultural sector.

Most studies on Nigerian agriculture and generally in agriculture that applied the stochastic frontier methodology found levels of production inefficiencies (Oni et al., 2009; Bravo-Ureta and Rieger, 1991; Battese, 1992; Bravo-Ureta and Pinheiro, 1993, 1997; Djokoto, 2012; Kea et al., 2016). The fact remains that producers' performance lies beneath the frontier level. The presence of inefficiency means that the technology output (Value added) can be increased without additional input. Hence, empirical measures of production efficiency are necessary to determine the levels of inefficiency and the magnitude of the possible reduction in inefficiency or the gains that could be obtained by improving performance in agricultural production or the agricultural sector. The Cobb-Douglas functional form remains widely used in literature, as exemplified by the works (Belotti and Ildardi, 2018; Bravo-Ureta and Pinheiro, 1993). A key advantage of the Cobb-Douglas specification is its ease of interpretation and comparability

across studies. Furthermore, its results can be used to approximate the more complex decompositions of productivity change, such as those proposed (O'Donnell, 2018). This makes the Cobb Douglas form a valuable tool for understanding and comparing productivity across different contexts and periods.

The research gap is to investigate the possibility of reducing production inefficiency through the output-input oriented routes or strategies. This might be more cost-effective by focusing on improving production efficiency using the same or reducing the input vector (Belbase and Grabowski, 1985; Bravo-Ureta and Pinheiro, 1997). Alternatively, reducing production inefficiency is a step in a process that might lead to substantial resource savings via the input-oriented strategy. These resource savings have important implications for both policy formulation and production management and have not been explored in the Nigerian agricultural sector (Bravo-Ureta Rieger, 1991).

Secondly, the time series nature/characteristics of the data set were not examined by many authors, and others (Karagiannis and Tzouvelekas, 2001; Djokoto, 2012). The current study attempts to fill this apparent gap. Hence, the study focused on the efficiency of agricultural production in the period 1960 to 2021, using output-oriented and input-oriented measures.

1.2 Objectives

The general objective is to apply the stochastic production function model to the Nigerian agricultural sector.

The specific objectives are to:

- estimate production efficiency/inefficiency in the sector,
- examine the potential for reduction in cost through improvement in production efficiency,
- calculating the returns to scale in the sector,
- Compare the production efficiency measures.
- Make policy recommendations based on the findings of this study.

1.3 Justification of the Study

The measurement of production efficiency, technical efficiency (TE), allocative efficiency (AE), economic efficiency (EE) and cost efficiency (CE) are areas of research in the developed and developing economies (Tadesse and Krishnamoorthy, 1497). This is so in emerging agrarian or agricultural economies. This is because in these agrarian economies, resources are meagre and opportunities for developing and adopting better technologies are dwindling (Ali and Chaudhury, 1990; Tadesse and Krishnamoorthy, 1997). Production efficiency studies can profit developing economies by assessing the potential for production increase and enhancing production efficiency. This neglected source of productivity

gains can be improved utilizing existing resources and current technology or by reducing input vector.

This notion tends to suggest that paying attention to and measuring productivity gains from a more efficient use of existing resources and technology is justified (Squires and Tobor, 1991; Bravo-Ureta and Pinheiro, 1993; 1997).

Most agricultural economics are characterised by production inefficiencies (technical, allocative, economic and cost inefficiency). The existence of such inadequacies in production efficiency indicates output can be increased without extra input or the calling for and adoption of new technologies. Hence, the empirical measurement of production efficiency is necessary to determine the magnitude of the gains that could be obtained by improving performance in the agricultural sectors of these nations with the available technology.

This study is of policy relevance. One, it provides an empirical measure of different production efficiency/inefficiency indices. Two, it examines the possibilities of cost savings and inefficiency reduction based on its empirical outcomes.

In methodology, EE is the product of TE and AE (Farrell's, 1957). Since it is recognised that EE is composed of TE and AE, it therefore stands to reason to assert that economic inefficiency arises from a combination of the technical and allocative components. While cost inefficiency arises from a combination of technical and allocative components, it arises from EE. It is of policy relevance to quantify their inefficiencies to indicate the need for improvement in efficiency, and possible cost savings based on the levels of the inefficiencies that are empirically determined.

Technical efficiency of input is a prerequisite for economic efficiency (Ikram et al., 2016). It simply involves a lack of waste in the input level used in production. Economic efficiency requires the greatest possible production from a given resource, that is, technical efficiency (fare and Loveli; 1978). In Nigerian agriculture, there is a need for recent studies and empirical evidence concerning production with two aims. One is to investigate the possibility of output increases with a constant bundle/vector of inputs. The second is to examine the possibility of a significant reduction in the vector of input for the same output level.

1.4 Production Function Analysis

Output is measured as production value. It is the worth of the overall quantity of goods and services in the agricultural industry. (Lipsey and Crystal, 1999). The production function, that is the Cobb. Douglas, will be used to calculate the output, while the elasticity and the Marginal Value Product (MVP) or the shadow price of each of the inputs used in production is also obtainable. The MVP/Shadow price in a perfectly competitive market, is the prices of inputs. It is used to measure resource-use efficiency of the sector. With a value-added dependent variable, the emphasis is on the interpretation of the variation in production value in terms of changes in the input (Mundlak et al., 2002).

1.5 Estimation Problem

This research uses a single equation approach; although its estimation in production function leads to unreliable estimates, this is because the analysis presumed immediate adjustment of output to changes in inputs level. This assumption brings about a simultaneity bias, this is a limitation (Zellner et al., 1966; Kalirayan and Flinn, 1983). Accordingly, this shortcoming can be resolved by assuming that vagueness wraps forthcoming output only, at the inception of the production, all prices, and the technical unit of production are known (Zellner et al., 1966). Producers optimises expected gains instead of the actual gains. In economics, the dependence of a variable on another usually requires time. Very often, the dependent variable responds to the independent variable over time (a lag time).

1.6 Variables

The sector is almost entirely rain-fed with labour as the main input. Rural-urban migration of able-bodied men is depleting the productive labour force (Raheem et al., 2014). The farming population is ageing, and labour is measured in man-days.

1.7 Fertiliser

Average fertiliser use is 13kg per hectare, while the world average is 100kg per hectare. In Asia, the average fertiliser use is 150kg per hectare. Hence, the rate of use of this input is very low, which is measured in kilograms (Daramola, 2014).

1.8 Land

In recent times, growth in agricultural output has generally come from

expansion in cultivated land. Production has moved into marginal lands. Hence, the potential for output increases from this factor is diminishing (Kienzle and Sims, 2015). The average farm size is 2.5ha, so the sector is largely a smallholder system (Daramola, 2014).

1.9 Tractors

The Nigerian agricultural sector has access to, on average, 10 tractors per 100 hectares compared with 240 tractors per 100 hectares in Indonesia. Not many farmers have access to this input in Nigeria (Daramola, 2014).

2. METHODOLOGY

2.1 Data collection

The study uses a secondary, annual time series data for the Nigerian agricultural sector, gathered from the Food and Agriculture Organisation (FAO) and the National Bureau of Statistics (NBS) database. Variables used in the study include values of agricultural gross domestic product (GDP) of the sector, fertiliser usage in kilograms, Agricultural labour in man-days and the number of tractors available in the Agricultural segment. The data set covers the period 1960-2021.

2.2 Method of Data Analysis

2.2.1 Model Specification

Following, a stochastic production function with the disturbance term composed of some parts, a systematic part (v) and a one-sided (u) component is specified for the agricultural sector of Nigeria (Aigner et al., 1977; Mausen and Van den Broeck, 1977).

The analytical framework borrows from the works of (Bravo-Ureta and Pinheiro, 1997; Karagiannis and Tzouvelekas, 2001).

The production function is specified as

$$V_{ait} = f(X_{1t}, X_{2t}, X_{3t}, X_{4t}) \tag{1}$$

Where: V_{ait} = production value of the sector, and

X_{it} 's = factors enhancing value added.

The Cobb-Douglas and stochastic production function used is given as:

$$V_{ait} = \beta X_{1t}^{b_1} X_{2t}^{b_2} X_{3t}^{b_3} X_{4t}^{b_4} e^{v_i - u_i} \tag{2}$$

Where: V_{ait} = yearly value added for the sector.

c = input vectors hypothesized in the sector.

Bis = vector of parameters to be estimated, and

v_i = two-sided normally distributed, systematic component. This captures random variations in V_{ait} due to factors outside the sector. It is assumed to be independently and identically (ILD) distributed as $Na(0, \sigma_v^2)$

u_i is the one-sided efficiency component with a half-normal distribution. It is assumed to have a non-negative distribution with $N \sim (0, \delta u^2)$ as a random variable that accounts for the existence of technical inefficiency. $V_i - U_i = \epsilon_i$, where ϵ_i represents the error term of the traditional deterministic production function formulation.

The estimating stochastic frontier production function (SFPF) equation, log-linearised is given as.

$$\ln V_{ait} = \ln \beta + b_1 \ln X_{1t} + b_2 \ln X_{2t} + b_3 \ln X_{3t} + b_4 \ln X_{4t} + V_i - U_i \tag{3}$$

The maximum likelihood estimates (MLE) of (3) on the assumption that V_i and U_i are independent provide estimators for the parameters b_i 's, the variance parameters for the one-sided U as σ_u^2 and for the two-sided V as σ_v^2 . The sum of these variances gives the σ^2 . Hence, $\sigma^2 = \sigma_u^2 + \sigma_v^2$.

The ratio of the two standard deviations of the error terms as used is called Lambda (λ) and $\lambda = \sigma_u / \sigma_v$ (Jondrow et al., 1982). While $\gamma = \sigma_u^2 / \sigma^2$. Lambda (λ) ≥ 1 and $0 < \gamma \leq 1$.

2.3 Stochastic Frontier Cost Function (SFCF)

Following Taylor et. al (1986), the analytically derived SFCF is represented as a Cobb-Douglas function as $C^* = f(P_{x1}, P_{x2}, P_{x3}, P_{x4}, V^*_{it})$ (4)

Where

P_{xit} 's are the average prices of the inputs and

V^*_{it} the value added for the t year adjusted for the stochastic noise captured by V_i , where $V_{it} = V^*_{it} + U_{it} - V_i$.

The multiplicative form of the SFCF is specified as

$$C^* = K P_{x1}^{a_1} P_{x2}^{a_2} P_{x3}^{a_3} P_{x4}^{a_4} e^{v_i + u_i} \tag{5}$$

The analytically derived cost equation is presented in the log form as:

$$\ln C^* = \ln k + a_1 \ln P_{xit} + a_2 \ln P_{x2t} + a_3 \ln P_{x3t} + a_4 \ln P_{x4t} + \theta \ln V_{it} + v_i + u_i \tag{6}$$

C^* is the minimum cost associated with the generation of V_{it} . Due to the self-dual nature of the SFPF and SFCF, for the duality theory to be upheld, all the parameters of the SFCF are derived from those of the SFPF (Taylor et. al., 1986; Jeffrey and Xu, 1998; Rahji, 2003; 2019).

2.4 Analytical Framework

2.4.1 Output-Oriented Measures of Production Efficiency-

- Estimate the SFPF to obtain $TE = \frac{Q}{Q^*}$
- adjusted output $Q^* = \frac{Q}{TE}$
- analytically derive the SFCF as

$$Inc^* = a_0 + a_1 \ln x_1 + a_2 \ln x_2 + a_3 \ln x_3 + a_4 \ln x_4 + \theta \ln Q^* \tag{7}$$

Where:

$$c^* = e^{a_0 + a_1 \ln x_1 + a_2 \ln x_2 + a_3 \ln x_3 + a_4 \ln x_4 + \theta \ln Q^*} \tag{8}$$

Given $\theta = \frac{1}{RTS}$; *RTS from the SFPF and*

- Generate c^* The lowest point of the cost function for each observation.
- Combine actual cost (c) with c^* to generate

$$EE = \frac{c^*}{c}; \quad C > c^* \text{ and } \frac{c^*}{c} < 1$$

This is the output-oriented EE

But $EE = AE \times TE$

Where:

$$AE = \frac{EE}{TE} \quad \text{Which is the output-oriented AE}$$

2.4.2 Input-Oriented Measures of Production Efficiency

Steps

First specify a SFPF $Q = f(x_1, x_2, x_3, x_4)$ Cobb-Douglas based \rightarrow

$$\#] [p'Q = A x_1^{B_1} x_2^{B_2} x_3^{B_3} x_4^{B_4} \tag{9}$$

$$\Rightarrow \ln Q = \ln A + B_1 \ln x_1 + B_2 \ln x_2 + B_3 \ln x_3 + B_4 \ln x_4 + v_i - u_i$$

- Estimate this function to obtain $TE = \frac{Q}{Q^*}$, The TE here, which is output-oriented, generates Q^*
- The Q^* = The analytically derived cost function
- The Q^* = The derived technically efficient input bundles
- From the analytically derived cost function, the economically efficient input bundles

2.5 Technically efficient input bundles

The technically efficient input bundles x_{it} are,

$$(i) Q^* = f(x, y) \text{ and } (ii) \frac{x_1}{x_i} = k_i, i > 1 \text{ simultaneously}$$

$$\text{But from (ii), } \frac{x_1}{x_2}, \frac{x_1}{x_3}, \text{ and } \frac{x_1}{x_4}$$

Solve simultaneously.

$$Q = f(x_1, x_2) \text{ and } Q^* = f(x_1/x_2)$$

$$Q^* = f(x_1, x_3) \text{ and } Q^* = f(x_1/x_3) \text{ and}$$

$$Q^* = f(x_1, x_4) \text{ and } Q^* = f(x_1/x_4)$$

The Cobb-Douglas specification for the simultaneous equation is.

$$Q^* = A x_1^{\gamma_1} x_2^{\gamma_2} \tag{10}$$

Since production is technically efficient ($TE = 1$) and since the efficiency parameter in a Cobb-Douglas function is A, then A = 1. The function becomes.

$$Q^* = 1 x_1^{\gamma_1} x_2^{\gamma_2}$$

$$\ln Q^* = \gamma_1 \ln x_1 + \gamma_2 \ln x_2 \tag{11}$$

The three pairs of simultaneous equations are.

$$\begin{aligned}
 x_{2j} &= f(x_1) && \text{for } i = 1, 2, 3, 4 \\
 x_{3j} &= f(x_1) && j = 1, 2, 3, 4, \dots, N. \\
 x_{4j} &= f(x_1)
 \end{aligned}$$

The matrix of technical input bundles (x_{it}) are,

$$\begin{bmatrix}
 x_{11} & x_{12} & x_{13} & x_{14} \\
 x_{21} & x_{22} & x_{23} & x_{24} \\
 \vdots & \vdots & \vdots & \vdots \\
 x_{N1} & x_{N2} & x_{N3} & x_{N4}
 \end{bmatrix}$$

2.6 Economically Efficient Input Bundles

Using Shephard's lemma, the economically efficient input bundles and the input demand functions from the analytically derived SFCF.

The Xies are from the input demand functions.

Theoretically, the demand for x_1

$$\frac{\partial c^*}{\partial p_i} = x_i \quad i = 1, 2, 3, 4$$

$$\text{But } Inc^* = a_0 + a_1 I_n p_1 + a_2 I_n p_2 + a_3 I_n p_3 + a_4 I_n p_4 + \theta I_n Q^* \tag{12}$$

$$\text{where } \frac{\partial I_n c^*}{\partial I_n p_i} = a_i \quad i = 1, 2, 3, 4.$$

$$\frac{\partial c^*}{\partial p_i}$$

Where,

$$\frac{\partial I_n c^*}{\partial I_n p_i} = \frac{\partial c^*}{\frac{c^*}{p_i}}$$

$$\frac{\partial I_n c^*}{\partial I_n p_i} = \frac{\partial c^*}{c^*} \times \frac{p_i}{\partial p_i}$$

$$= \frac{\partial c^*}{\partial p_i} \cdot \frac{p_i}{c^*} \simeq a_i$$

$$\therefore \frac{\partial c^*}{\partial p_i} = \frac{a_i c^*}{p_i} = x_i; \quad i = 1, 2, 3, 4$$

Given the a_i s input: elasticity of cost c_i from the SFCF and p_i From the raw data.

$$\frac{\partial c^*}{\partial p_i} = \frac{a_{ij} c^*}{p_{ij}}$$

The matrix of the economically efficient input bundles (x_{ie}) is.

$$\begin{pmatrix}
 \frac{a_{11}c^*}{p_{11}} & \frac{a_{12}c^*}{p_{12}} & \frac{a_{13}c^*}{p_{13}} & \frac{a_{14}c^*}{p_{14}} \\
 \frac{a_{21}c^*}{p_{21}} & \frac{a_{22}c^*}{p_{22}} & \frac{a_{23}c^*}{p_{23}} & \frac{a_{24}c^*}{p_{24}} \\
 \frac{a_{31}c^*}{p_{31}} & \frac{a_{32}c^*}{p_{32}} & \frac{a_{33}c^*}{p_{33}} & \frac{a_{34}c^*}{p_{34}}
 \end{pmatrix}$$

$$\frac{a_{ij}c^*}{p_{ji}} \quad \frac{a_{j2}c^*}{p_{j2}} \quad \frac{a_{j3}c^*}{p_{j3}} \quad \frac{a_{j4}c^*}{p_{j4}}$$

The original input bundles (x_{i0}) are known.

2.7 Calculation of the Production Efficiency Measures

Multiply the original (o) by the input prices matrix. The technically efficient input bundles (x_{i0}) and the economically efficient input bundles (x_{ie}) to obtain,

$$TE = \frac{\sum x_{ie} p_i}{\sum x_{i0} p_i} \tag{14}$$

$$EE = \frac{\sum x_{ie} p_i}{\sum x_{i0} p_i} \tag{15}$$

But AE = EE/TE

$$\text{Hence AE} = \frac{\sum x_{ie} p_i}{\sum x_{i0} p_i} \tag{16}$$

These are the input-oriented measures of production efficiency

2.8 Unit Root Test Regression Variables

2.8.1 Unit Roots Test

This study used annual agricultural sector data from 1960 to 2021. The augmented Dickey-Fuller (ADF) test checks the variables for stationarity among the variables in the model.

Cointegration Analysis The initial test to avoid spurious regression result is the Cointegrating Regression Durbin-Watson (CRDW) and Engle-Granger/augmented Engle-Granger (EG/AEG). The regression EG/AEG test affirms that if the deviations from the I(1) variables in a regression are found to be I(0), then linear combination of the variables cancels out the stochastic trends. Such variables are meaningful and not spurious. Thus, they have a long-term relationship. The traditional regression approach can be applied to the non-stationary time series. It is therefore known as a cointegration regression. While the slopes are known as the cointegrating parameters. This is a significant contribution of the concepts of unit root, cointegration and others to regression analysis (Gujarati, 2003).

2.8.2 Johansen Cointegration Test

As evaluates the association between variables, where each variable has a series that are not stationary (Johansen's, 1988; Dickey, Jansen and Thornton, 1991). It examines the existence of numbers of cointegrating vectors amid the integrated variables (Harris, 1995).

2.8.3 Cointegration and Regression Analysis

In regression analysis, a unit root indicates that the variables are not stationary at level I (0) The ADF test, which is a univariate test is normally used to confirms a unit root or otherwise. Nonetheless, if the residual from the non-stationary variables is I (0), then the linear combination of the non-stationary variables will be I (0). meaning that the linear combination of variables cancels out the stochastic trends in them. Thus, a regression of the unit root time series variables on one another would be meaningful, reasonable, not spurious, and non-nonsensical. In this case, the unit root variables are cointegrated; they thus have a long-term or equilibrium relationship among them and are stationary. Hence, the traditional regression methodology applies to the non-stationary time series data based on the outcome of the test on the residuals.

The Johansen cointegration test is used to investigates the relationship among groups of variables where each variable has a unit root. (Dickey, Jansen, and Thornton, 191). To do that, non-stationary variables must be in the same order of I (1).

Thus, test is based on the variables being I (1) (Johansen, 1988). When these conditions are met, the test uses the I (0) variables to confirm the existence of and the number of cointegrated equations amongst the integrated variables (Harris, 1995). This confirmation means that the variables have the tendency to revert to the average. These approach validates integration, stationarity of the variables used, and cointegration.

3. RESULTS AND DISCUSSION

3.1 Unit Root Test Results

Table 1 presents the results of the unit root test for each variable in the model, using the Augmented Dickey-Fuller (ADF,1979) at the 5% significance level. The results showed all series were I (1). They exhibit a stochastic trend.

Table 1: Unit Root Test		
Variables	Level	Difference
LnQ	-2.6183	-4.3988*
LnX ₁	-2.5469	-3.9581*
LnX ₂	-1.2845	-3.5366*
LnX ₃	-2.9413	-5.0065*
LnX ₄	-1.8352	-4.1507*

Source: Author's computation

The series is order I(1), the best lag length should be the Johansen cointegration test. It shows long-run relationships among the variables. The unit root test rejects the null hypothesis at the 5% level for the first difference. These results indicate that the series is of order I (1).

3.2 VAR Lag Order Selection Test

This study uses two lag length as indicated by the test, to evaluate the cointegration between the variables

Table 2: VAR Lag Order Selection Criterion Endogenous variable. LNQ LNX1 LNX2 LNX3 LNX4 Exogenous variable C

Lag	LogL	LR	FPE	AIC	SC	HQ
0	170.4555	NA	1.19e-09	-6.363675	-6.176055	-6.291746
1	530.5174	637.0325	3.02e-15	-19.25067	-18.12495*	-18.81909
2	565.8514	55.71905*	2.08e-15*	-19.64813*	-17.58432	-18.85691*
3	576.6639	14.97116	3.86e-15	-19.10246	-16.10054	-17.95160
4	596.7476	23.94593	5.37e-15	-18.91337	-14.97336	-17.40286

Source: Author’s computation (2022)

Indicates lag order selected by the criterion.

LR sequential modified LR test statistics (each test at 5% level)

FPE: Final prediction error

AIC Akaike information criterion

SC Schwarz information criterion

HQ Hannan-Quinn information criterion

Endogenous variables: LNGDP, LNfert, LNlabour, LNland, LNtract.

Exogenous variables: C

3.3 Unrestricted Cointegration Rank Test

The unrestricted trace tests show the possibility of co-integrating vectors with trace statistics, providing compelling support for the long-run connection. Table 3 shows one co-integrating vector(s) with trace statistics, which is strong evidence for the long-run relationship among the variables of the model.

Table 3: Unrestricted cointegration rank test (Trace)

Hypothesised		Trace	0.05	
No. of CE(s)	Eigen value	Statistic	Critical Value	Prob.**
None*	0.478336	79.72656	69.81889	0.0066
At most 1	0.369924	45.23777	47.85613	0.0863
At most 2	0.182022	20.75632	29.79707	0.3730
At most 3	0.151146	10.10759	15.49471	0.2725
At most 4	0.026484	1.422600	3.841466	0.2330

Source: Author’s computation, 2022.

The trace test indicates one cointegrating equation at the 0.05 level.

*Denotes rejection of the hypothesis at 0/05 level

**Mackinnon-Haug Michells (1999) p-values

3.4 Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

The four co-integrating vectors in the model are correlated in the long term as confirmed by the test. The coefficients are discussed in the paper.

Therefore, the regression results are statistically valid, and allows for further analysis with the available data.

Table 4: Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesised		Max-Eigen	0.05	
No. of CE(s)	Eigenvalue	Statistic	Critical Value	Prob.**
None *	0.478336	34.48880	33.87687	0.0422
At most 1	0.369924	24.48145	27.58434	0.1188
At most 2	0.182022	10.64872	21.13162	0.6822
At most 3	0.151146	8.684995	14.26460	0.3133
At most 4	0.026484	1.422600	3.841466	0.2330

Source: Author’s computation, 2022.

The max-eigenvalue test indicates one cointegrating equation(s) at the 0.05 level.

*Denotes rejection of the hypothesis at the 0.05 level

**Mackinnon-Haug Michelis (1999) p-values

3.5 The Estimated Stochastic Frontier Production Function

Table 5 contains the result of the estimated stochastic frontier production function (SFPF). The lambda (λ) is the ratio of the standard deviations of u_2 (u_2). The standard deviation has a value of 1.8853, which is greater than one. This signifies a good fit for the estimated model. It also indicates the appropriateness of the required distributional assumptions for the decomposed error term. The result has a large sigma (σ) value of 0.3537 which is relatively large and significant at the 5% level. (Tadesse and Krishnamoorthy, 1997).

The value of the coefficients of the SFPF are positively sign, indicating a direct relationship between the inputs and the output, and an expected range of between zero and one. The estimated variance of the one-sided error terms (u_2) is 0.0976, the statistical noise (v_2) is 0.0275. The output-oriented technical efficiency (γ) is 0.7802. This value is the ratio of the variance of the one-sided error term to the sum of the two variances (2). The technical inefficiency level is therefore 0.2198. The agricultural sector is 22% technically inefficient, and 78% output-oriented, technically efficient. The 22% reveals that the industry output can increase, improved technical efficiency without enhancing the input. This result supports studies with efficiency score of 0.60 - 0.85 in agriculture and food production in developing countries. For details see; who report an average TE of 47%, implying the sector is quite inefficient, with a 53% scope for improvement; evaluated average level of efficiency in different countries and found 72% in Ethiopia, Indian and Tanzania, 78% in Bangladesh, and 65% in Nigeria, respectively (Diaz et al., 2025; Asogwa et al., 2019). As reported that the overall mean technical efficiency of rice farming is 77% which implies that, on average, farm households have the potential to increase their rice production by 23% given the same level of inputs and technology (Ho and Shimada, 2019). Revealed that a mean technical efficiency index (TEI) of 0.72 was achieved, implying that output from urban agriculture production could be increased by 28% using available technologies (Mwajombe and Mlozi, 2015). As find that the mean technical efficiency level of tilapia farmers is 78% thus, the farmers operate 22% below the frontier production (Alam et al., 2012). The returns to scale were 0.9226, X_2^{cal} were 9.61, while the X_2^{tal} is 3.841 at the 5% level. So, accept the alternative hypothesis at 5% level, that $Eai \neq 1$. In this case, it is less than 1at 0.9226, indicating decreasing returns to scale. In their study on firm efficiency and returns to scale in layer production among smallholders in Jos-North, Nigeria, found returns to scale of 0.654; thus, $\sum \rho < 1$, which indicates a decreasing return to scale (Onuwa et al., 2022).

Table 5: The Estimated Stochastic Frontier Production Function

Variable	Coefficient	Std. Error	Z	P/Z/.
In X_{1t} (fert)	0.2634	0.0960	2.7438	0.0000
In X_{2t} (labr)	0.3159	0.0551	5.7332	0.0000
In X_{3t} (land)	0.1846	0.0896	2.0603	0.0211
In X_{4t} (Tract)	0.1587	0.0798	1.9887	0.0524
C	2.6375	1.2478	2.1137	0.0203

Source: Author’s computation, 2022

$\ln \sigma_v^2 = -3.5936$

$\ln \sigma_u^2 = -2.3269$

$\sigma_v = 0.1657$

$\sigma_u = 0.3124$

$\sigma_v^2 = 0.1251$

$\sigma = 0.3537$

$\lambda = 1.8853$

RTS = 0.9226

$\sigma_v^2 = 0.0275$

$\sigma_u^2 = 0.0976$

$\sigma^2 = 0.1251$

$\gamma = 0.7802$

3.6 Analytically Derived Stochastic Frontier Cost Function

Table 6 presents the result of the analytically derived stochastic frontier cost function for the sector. On the average, the inputs costs, production cost and the frontier cost (C*) values were all generated from the time series data. The EE of the sector is 0.5843, implying 58% economically efficient while the economic inefficiency level is 0.4157, indicating 42% inefficient. The estimated mean EE of 58% signify the ratio of least cost to real cost of production. This implies that significant cost savings of 42% in the sector can improve the other two production inefficiencies (TE and AE). These findings support on technical, allocative, and economic efficiency among smallholder maize farmers in Southwestern Ethiopia (Sisay, 2015). The results show that the technical, allocative, and economic efficiency scores were 62.3%, 57.1% and 39%, respectively. On estimation of technical, economic and allocative efficiencies in Sugarcane Production in South Africa, found mean technical, allocative and cost efficiency estimates are 68.5%, 61.5% and 41.8% respectively Cost efficiency (CE) which is the real/actual cost to the equivalent least /frontier cost given existing technology, is 1.7115 (Londiwe, 2014). This is the same because it is the inverse of the economic efficiency (EE). Implying that, on average, the sector accumulate costs that are 71% of the production cost compared to the best-practice year. From an aggregate and time series perspective, the best practice frontier is the potential value added (output) for the best practice year (Djokoto, 2012).

Variable	Coefficient	Std. Error	Z	P/Z/
P Fert	0.2855			
P Labr	0.3424			
P Land	0.2001			
P tract	0.1720			
Adj. output(θ^*)	1.0839			
Const.	-1.5659			

Source: Author’s computation,2022

TE = 0.7802

EE = 0.5843

AE = 0.7489

CE =1/EE

CE =1.7115

3.7 Input-Oriented Measures of Production Efficiency

The input-oriented technical efficiency (γ) of 0.7363 is the ratio of the cost of the technically efficient input bundle [Xe1Pi] to the rate of the actual observed operating inputs combination in the sector (Xa1 Pi). The input-oriented technical inefficiency is thus 0.2547. The agricultural sector is 74% input-oriented technically efficient, and 26% input-oriented technical inefficient. As found in their study on the technical efficiency of Nigerian agriculture that Nigeria is 72% technically efficient, suggesting that there is still room for improvement in the efficiency of Nigerian agriculture (Ogundari and Brümmer, 2011). The 26% inefficiency level implies the rate at which the value addition in the sector is enhanced without a proportionate decrease in input vector. On average, reducing input quantities proportionally can lower total costs by 26% without changing the sector’s total value added.

RTS is 0.9226, confirmed as X_{2cal} (9.61) exceeds X_{2tab} (3.841). The sector is thus characterised by decreasing returns to scale.

The input-oriented economic efficiency (EE) of 0.5385 is the ratio of the cost of the economically efficient input bundle (Xe1Pi) to the price of the actual operating input combination (Xa1Pi). The input-oriented economic inefficiency is 0.4615. The sector is 54% input-oriented and economically efficient, and 46% inefficient. The estimated mean input-oriented (EE) represents the ratio of the minimum to the actual cost in the sector. Improving technical and allocative efficiency could reduce costs by 46%. Theoretically, economic inefficiency is entirely due to input-oriented technical and allocative inefficiency.

Input-oriented cost efficiency, which is the inverse of EE, is 1.8570. This indicates an input-oriented cost inefficiency level of 0.8570. This suggests production costs could drop by 86% without reducing the sector’s value added. The input-oriented allocative efficiency is 0.7216, while allocative inefficiency measures 0.2784. This means the agricultural sector achieved a good allocation of the existing resources. However, the mean input

allocative efficiency of 0.7216 is smaller than its corresponding input-oriented technical efficiency of 0.7802. These results imply that the sector did better in achieving the maximum attainable output for given inputs than in allocating existing resources.

Input oriented	Output oriented
Technical efficiency (TE) 0.7463	Technical efficiency (TE) 0.7802
Technical inefficiency (TI) 0.2547	Technical inefficiency (TI) 0, 2198
Allocative efficiency (AE) 0.7216	Allocative efficiency (AE) 0. 7489
Allocative inefficiency (AI) 0.2784	Allocative inefficiency (AI) 0. 2511
Economic efficiency (EE) 0 5385	Economic efficiency (EE) 0 5843
Economic inefficiency (EI) 0 4615	Economic inefficiency (EI) 0 4157
Cost efficiency (CE) 1.8570	Cost efficiency (CE)1.7115
Cost inefficiency (CI) 0.8570	Cost inefficiency (CI) 0.7115

Source: Author’s computation, 2022.

*Letter I indicates inefficiency

4. CONCLUSION AND RECOMMENDATION

The study concludes that the output-oriented and input-oriented technical efficiency are equal under constant returns to scale. The output-oriented technical efficiency is greater than the input-oriented TE under decreasing returns to scale, and the output-oriented technical efficiency is less than the input-oriented technical efficiency under increasing returns to scale.

The results, as presented in Table 7, indicate that output-oriented TE, AE, and EE are greater than their corresponding input-oriented TE, AE, and EE. Producing under an output-oriented route is better than under an input-oriented strategy, as the measures are characterised by higher production efficiency. On the other hand, input-oriented efficiencies and inefficiencies are greater than their corresponding output-oriented efficiencies and inefficiencies. TI, AI, and EI under the input-oriented route are indicative of higher production inefficiency. Second, both CE and CI are higher under the input-oriented method than under the output-oriented strategy. These findings confirm that the output-oriented route is preferable to its counterpart in any attempt at production adjustments in the sector.

The following recommendations are made based on the results of this study. The sector should utilise available inputs to optimise output, the technology in the industry should be improved to contribute to achieving better outcomes, and the sector should employ an input-oriented, cost-reduction approach.

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